



**CALIFORNIA STATE SCIENCE FAIR
2005 PROJECT SUMMARY**

Name(s) Olga V. Mandelshtam	Project Number S1209
Project Title A Study of the $3x+1$ Transformation and Its Continuous Limit	
Abstract Objectives/Goals My experiment is an application of a novel method to a century-old problem. The Collatz (or $3x+1$) problem deals with a simple sequence of iterations of a function $f(x)=3x+1$ for odd x , and $f(x)=x/2$ for even x . The yet unproven Collatz conjecture states that any sequence of iterations of $f(x)$ will eventually run to 1. In my project I developed a circle map that represents the Collatz transformation and investigated its properties to make progress in the problem. I expected that my method would have much potential in studies of the Collatz problem, and much of interest could be found in investigations such as this one. Methods/Materials I wrote numerous programs to explore various aspects of the problem I was investigating. Each of my programs stemmed from a particular aspect that interested me for further study. Results I studied and proved some properties of this map, and then related it to a continuous transformation. I then studied iterations for a family of continuous maps of this type, both numerically and analytically. My most interesting results are related to the pattern of the slopes of those iterations. Conclusions/Discussion I found that the pattern of slopes demonstrates a striking sequence of pseudoperiods that are related to my numerical calculation of an exact formula for the rotation number. I also related certain properties of iterations of this continuous transformation to the original Collatz problem. The maps discussed in this paper have potential to be of use in future work in the area.	
Summary Statement I developed a novel approach to the Collatz problem by representing the $3x+1$ transformation as a circle map, and studying the properties of it and its continuous limit.	
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