CALIFORNIA STATE SCIENCE FAIR
2006 PROJECT SUMMARY

Name(s)

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Project Number

J1212

Project Title

Divisibility Discovery: A New Divisibility Rule

Abstract

Objectives/Goals

Different divisibility rules for some specific numbers have been established. But is there a general divisibility rule, or pattern, that applies to any number? I hypothesized that there is a general divisibility rule for any divisor ending in 1, 3, 7, and 9. The reason I chose 1, 3, 7, and 9 is because divisors ending in 0, 2, 4, 5, 6, and 8 can be broken down into divisors ending in 1, 3, 7, or 9 unless they are powers of 2 or 5.

Methods/Materials

I used 11, 21, 31, and 41 as divisors ending in 1 and chose some of their multiples as dividends. I studied the relationship between the digits of the dividends and divisors and performed different operations on the digits to find the operation that would always produce results that are multiples of the divisors. This operation would be the divisibility rule for divisors ending in 1. I established the rules for divisors ending in 3, 7, and 9 in the same way. Then I used Microsoft Excel to test my rules with greater dividends and divisors.

Results

For any dividend, 10A+a(1), where a(1) is the unit's digit of the number, and A is the number formed by the dividend without the unit's digit, and any divisor, 10B+b(1), where b(1)=1, 3, 7, or 9, and B is the number formed by the divisor without the unit's digit, the divisibility rules for divisors ending in 1, 3, 7, and 9 are A-a(1)*B, A-a(1)*(7B+2), A-a(1)*(3B+2), and A-a(1)*(9B+8) respectively, which means for divisors ending in 1, if A-a(1)*B is divisible by the divisor, the original dividend is also divisible by the divisor, and the same for divisors ending in 3, 7, and 9. For example, is 5082 divisible by 231? In this case A=508, a(1)=2, B=23, and b(1)=1. Since 508-2*23=462, and 462 is divisible by 231, 5082 is divisible by 231. These rules were valid for every dividend and divisor tested using Microsoft Excel. Using modular arithmetic, I further proved the rules to be valid for any number.

Conclusions/Discussion

My hypothesis is supported because the results show that there is a general divisibility rule for divisors ending in 1, 3, 7 and 9, and the rule is related to A, a(1), and B. The rules I established contribute to the number theory and can be applied to prime number testing, which is important in fields such as cryptography. Next, I will try to find a more general divisibility rule for any number ending in any digit.

Summary Statement

There is a general divisibility rule for any divisor ending in 1, 3, 7, or 9 and the rule is related to A, a(1), and B.

Help Received

Mrs. Diana Herrington gave advice on report. Mom designed computer program to test my rules. Grandpa helped glue board.