CALIFORNIA STATE SCIENCE FAIR
2006 PROJECT SUMMARY

Name(s)

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Project Number

J1218

Project Title

The Nimber-Simplex Graph as a Model to Compare LDPC and Turbo Codes

Abstract

Objectives/Goals

The objective of this project was to expand on my previous projects, which defined and characterized the Nimber-Simplex graph. The first goal was to prove that all binary linear codes map to the graph. The second goal was to demonstrate the graph's practical applicability by comparing low density parity check (LDPC) codes and turbo codes.

Methods/Materials

To prove my first hypothesis, I formally defined the Nimber-Simplex graph, then proved that all binary linear codes map to the graph by mapping message digits to the vertices. I showed examples using Hamming codes, single parity check codes, and maximum length codes. Next, I formally defined binary linear sets as abelian groups with self-inverse and proved that they map to the graph. I showed examples using BCH codes and polynomials in binary fields. To prove my second hypothesis, I showed how Gaussian elimination can be used to derive standard generator and check matrices from an LDPC matrix. Using these standardized matrices, I mapped the LDPC (15, 7)-code to the graph and then showed a construction of standard parity check codes and turbo codes which are roughly equivalent to the original LDPC code. I briefly discussed measures of code performance and outlined a direct comparison between the two families, proving my hypothesis.

Results

This project proves its hypotheses and gives an outline for future study. The proof that all binary linear codes and sets map to the graph allows a direct comparison between two highly effective but dissimilar modern code families: LDPC codes and turbo codes. A proposal for implementation of this comparison is outlined.

Conclusions/Discussion

In this project, the Nimber-Simplex graph, which had been previously described as an abstract mathematical object, is shown to have applications to modern coding theory. The comparison between LDPC and turbo codes establishes a methodology to compare other code families. The Nimber-Simplex graph may be used to design new LDPC codes by reversing Gaussian elimination. The ability to map binary linear sets to the graph may also offer new ways of designing linear codes with a wide variety of properties.

Summary Statement

This project proves that all binary linear error-correcting codes map to the Nimber-Simplex graph and uses this mapping to formulate a direct comparison between LDPC and turbo codes.

Help Received

My father helped me learn the advanced coding theory necessary to create this project. Both of my parents assisted with backboard construction and reviewed the report for readability and technical accuracy. Dr. Duncan Buell provided valuable comments regarding my projects from both this and prior years.