



**CALIFORNIA STATE SCIENCE FAIR  
2007 PROJECT SUMMARY**

<b>Name(s)</b> Nathaniel C. Willy	<b>Project Number</b> <b>J1320</b>
<b>Project Title</b> <b>A Mathematical Proof of a Relationship between Fibonacci and Lucas Numbers</b>	
<p style="text-align: center;"><b>Abstract</b></p> <p><b>Objectives/Goals</b> Prove that <math>[2L(n)+L(n-3)]/5</math> equals a Fibonacci number for all values of <math>n</math>. In this expression, <math>n</math> is a sequential number in a series, and <math>L(n)</math> is the <math>n^{\text{th}}</math> Lucas number.</p> <p>The equation above is a more complex relationship between Fibonacci and Lucas numbers posed as a challenge problem in The Fibonacci Quarterly.</p> <p><b>Results</b> I proved that the stated equation relating Fibonacci and Lucas numbers was true using mathematical equations and algebraic manipulations. I also showed that the equation was valid using a chart populated with the Fibonacci and Lucas series numbers then graphically displayed results of the equation. In addition, I validated it in Microsoft Excel by plugging the appropriate Lucas numbers into this equation, and showed that it was true for the first 100 values of <math>n</math>.</p> <p>I also demonstrated some more simple relationships between Fibonacci and Lucas numbers along with their numerical patterns and matching mathematical equations.</p> <p><b>Conclusions/Discussion</b> Fibonacci numbers are a fascinating and famous series of numbers. The occurrence of the Fibonacci series is found throughout nature. There are also countless mathematical relationships between Fibonacci and Lucas numbers. While researching Fibonacci and Lucas numbers I became intrigued with these relationships and chose to focus my study on a mathematical analysis of these relationships. I thoroughly enjoyed this very interesting project.</p>	
<b>Summary Statement</b> A mathematical proof of a relationship between Fibonacci and Lucas numbers using algebra.	
<b>Help Received</b> My father and my algebra teacher, Mr. Merle, went over my proof to make sure I didn't make any errors.	