CALIFORNIA STATE SCIENCE FAIR
2008 PROJECT SUMMARY

Name(s)

Abraham P. Karplus

Project Number

J1308

Project Title

Self-Similar Sierpinski Fractals

Abstract

Objectives/Goals

The goal of my science fair project is to better understand self-similar objects and their fractal dimension. A self-similar object is one that is created out of multiple smaller copies of itself. A part of studying these fractals is understanding how to create them. Fractals do not have integer dimensions like a square or cube do. For instance, the dimension of a fractal triangle (known as a Sierpinski Triangle after its discoverer) is approximately 1.58. My investigative question is #How does fractal dimension vary in regular Sierpinski polygons as the number of sides of these polygons increases? Also, what is the limit of fractal dimension as the number of sides goes to infinity?# Based on the results for fractal triangles and squares, my hypothesis was: #The fractal dimension will increase as the number of sides increases.#

Methods/Materials

I wrote multiple programs that generate fractals. Some of these used recursive code, others random code (known as the Chaos Game). The programs were written in Scratch, a graphical programming language available free from MIT. In order to understand fractal dimension, I had to understand scaling factor, namely how many times bigger a given copy is than the next size smaller. I used trigonometry to derive a summation formula for the largest possible scaling factor such that the copies do not overlap for regular polygons. I also derived a closed-form version. The scaling factor is one of the two essential numbers (along with number of sides) needed to compute fractal dimension. I derived the formula for fractal dimension. I used my closed-form formula in Gnuplot to show my results. I used Maple to determine the limit of the closed-form equation.

Results

My computational results were that fractal dimension has a downward trend as the number of sides increases, though it does increase occasionally. Here are some of the formulas I derived: (n is the number of sides)

Summation Scaling Factor: s=2(1+cos(360/n)+cos(720/n)+cos(1080/n)+...) while the cosine term is positive

Closed Form Scaling Factor: s= 1 + sin((180+360*floor((n-1)/4))/n) / sin(180/n)

Fractal Dimension: f=log(n)/log(s)

Conclusions/Discussion

I have found that fractal dimension goes up and down with the number of sides, though mainly down. It approaches 1 when the number of sides goes to infinity.

Summary Statement

My project is mainly deriving formulas for scaling factor and fractal dimension of self-similar polygonal fractals.

Help Received

Father taught me trigonometry and provided access to Maple and gnuplot. Both parents helped type report.