| Name(s) | Project Number |
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| **Kaavya N. Jayram** | **S1606** |

| Project Title |
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| **Crank 0 Partitions and the Parity of the Partition Function** |

## Abstract

**Objectives/Goals**
The formal properties of integer partitions have been investigated for over 200 years by some of the brightest minds in mathematics such as Euler, Hardy, and Ramanujan, with surprising applications to modern physics and computer science. The partition function $p(n)$ denotes the number of ways in which an integer n can be written as an (unordered) sum of other integers. Motivated by Ramanujan's investigations into the modular properties of $p(n)$, this project aims to make progress on the parity problem of $p(n)$ by means of deriving generating functions for cranks and ranks.

**Results**
Berkovich and Garvan (2002) showed that there is always a bijection between the crank k and crank -k partitions of n for every $k>0$. Consequently, the parity problem for $p(n)$ reduces to studying crank 0 partitions. I obtained the following results:

(1) I derived a generating function for crank 0 partitions of n, which is similar to a generating function for $p(n)$. I also obtained a general form for the crank k generating function.

(2) I described an involution on crank 0 partitions of n, whose fixed points are called invariant partitions. I then derived a generating function for crank 0 invariant partitions.

(3) Finally, I derived a generating function for rank 0 self-conjugate partitions.

**Conclusions/Discussion**
The proof techniques are based on identifying and manipulating the key combinatorial objects underlying cranks and ranks, and avoid the analytic techniques inherent in previous methods.

A paper describing the above results has been accepted to the International Journal of Number Theory.

| Summary Statement |
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| I derived generating functions for the crank and rank of an integer partition to make progress on the parity problem for integer partitions. |

| Help Received |
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| Dr. Laurens Gunnarsen was my mentor for the project. |