CALIFORNIA STATE SCIENCE FAIR
2010 PROJECT SUMMARY

Name(s)

Ivan R. Pyzow

Project Number

S1613

Project Title

Invisible Infinities: Determining the Fraction of Lattice Points Visible from the Origin in the Third Dimension

Abstract

Objectives/Goals

The objective was to determine the fraction of "visible" and "invisble" lattice points from the origin out of the entire set of lattice points in an infinite three dimensional space. I hypothesized that it would involve pi^3, because the known fraction of visible points in the second dimension is 6/pi^2.

Methods/Materials

I examined this problem in the second dimension to give insight into the third. After constructing a sample grid of 10x10 and manually determining visible and invisible point by drawing lines (representing lines of sight) from the origin to those points, I was able to recognize patterns which allowed me to contruct larger graphs with more efficiency. I was then able to construct 3D grids and plot visible points using patterns recognized in two dimensions. I created various representations of three dimensional graphs which I analyzed for patterns which might yield methods for approximating the fraction of visible points.

Results

I discovered that 2D visible points have coordinates (a,b) where a and b are relatively prime. In 3D, visible points have cooridinates (a,b,c) where a, b, c are relatively prime. I was able to reason that the fraction of visible points is 1/(riemann zeta of 3) ~ 0.83 because of the knowledge that the probability of m numbers being relatively prime is 1/(riemann zeta of m). By recognizing the grid of "invisible" points as the summation of an infinite number of grids with patterns, I was able to fashion an approximation of the fraction of invisible points with a series. I compared my series to 1/(riemann zeta of m) for the 2nd, 3rd, and the mth dimension and evaluated the error.

Conclusions/Discussion

My hypothesis was wrong. The fraction that I achieved was 1/(riemann zeta of 3), which is irrational, and based on my research of literature, does not have a representation involving pi^3. However, I am now able to hypothesize that the fraction of visible points in the mth dimension will be 1/(riemann zeta of m). This also suggests (by evaluating the fraction) that in the mth dimension as m goes to infinity, all points would be visible.

Summary Statement

My project deals with methods of determining the fraction of visible lattice points from the origin in the third dimension and their relation to prime numbers, the Riemann Zeta function, and other infinite series.

Help Received

My project advisor suggested books and concepts to research