CALIFORNIA STATE SCIENCE FAIR
2013 PROJECT SUMMARY

Name(s)

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Project Number

S1407

Project Title

New Methods of Generating Sets of Intransitive Dice

Abstract

Objectives/Goals

Although there are many well-known sets of intransitive dice, there are very few published methods for generating these sets, and very little published work that explains why these sets are intransitive. Our project responds to an article published by M.N. Deshpande in 2000, which outlines one such method. The objective of our project was to explore new methods of generating sets of intransitive dice, given a set of specific dimensions and allowing no repeated face values within sets.

Methods/Materials

Initially, we used an algorithm coded in the computing software Wolfram Mathematica 9 to find all possible intransitive sets of 3 dice with 3 faces each, using face values of 1-9. We then identified a pattern underlying each set, and attempted to expand this pattern to fit larger sets. After identifying each working pattern, we searched for a proof that demonstrates the reasons that the pattern generates intransitive sets, and proves that the pattern will generate intransitive sets of any size. We then experimented with systematically altering the newly generated sets.

Results

The algorithm discovered all 5 possible sets that fit our criteria. One of these sets had already been studied in Deshpande's article. Manipulating the other 4 led to the discovery of 3 new patterns that generate sets of intransitive dice. We were able to prove that these patterns yield intransitive sets by developing algebraic expressions for the probability of victory of each die over its neighbors. Sets altered by switching adjacent integers or removing a row of faces remained intransitive, while adding additional faces or dice proved unreliable.

Conclusions/Discussion

Based on the success of our experimentation, we conclude that there exist at least 3 other methods for generating sets of any size besides the method published by Deshpande. This supports our initial hypothesis. Our project expands not only on the work of M.N. Deshpande (2000), by creating similar methods of generating sets, but on the work of Schaefer and Schweig (2012) by providing additional proofs that intransitive sets of any size can be generated.

Summary Statement

Our experiment responds to a question posed by Indian mathematician M. N. Deshpande, and encompasses 4 previously undiscovered methods (with accompanying proofs) of generating sets of intransitive dice.

Help Received

Dr. Brian Conrey helped write algorithm, taught us how to use computational software.