CALIFORNIA STATE SCIENCE FAIR
2014 PROJECT SUMMARY

Name(s)

Nitya Mani

Project Number

34471

Project Title

Characterizing the n-Division Points of Genus-0 Curves through Straight Edge and Compass Constructions

Abstract

Objectives/Goals

This project explores the field of constructibility of n-division points (points dividing a closed curve into n pieces of equal arc length) through straightedge and compass constructions, by furthering the work done by the two major theorems in this field - the Gauss-Wantzel Theorem and Abel's Theorem on the Lemniscate. Three major problems in this field were investigated: determining a closed form solution for the regular polygons that can be constructed with a straightedge, compass, and trisector; finding the values of n such that the n-division points of the tricuspoid can be constructed with a straightedge and compass; and seeking a generalization of Abel's Theorem to the entire family of Serret curves (a family of curves including the lemniscate whose arc lengths share properties).

Methods/Materials

Two major mathematical fields were used to algebraically represent the problem of constructibility. Field theory was used to characterize the figures, angles, and lengths that are constructible under a given set of conditions, and the theory of elliptic integrals was used to determine expressions for the n-division points of these curves. Three areas of mathematics were used in the proofs of the theorems. Galois theory was used to find a solution to the first problem, algebraic geometry was used to relate geometric problems of constructibility and n-division points to algebraic ones, and complex analysis was used to examine the elliptic integrals that characterized the arc length of these curves, particularly when solving the third problem.

Results

Three major results were obtained through the research. A closed-form solution for the values of n for which the n-division points of a circle can be constructed with a compass, straightedge, and trisector was found; a theorem was proved that for all integer n, the n-division points of the tricuspoid curve are constructible; and it was determined that with a compass and a straightedge, arbitrary arc lengths on any Serret Curve can be added, subtracted, and multiplied.

Conclusions/Discussion

These results represent the product of a year of investigation, however, additional work is being done to explore related problems in this field, such as examining the n-division points of a circle constructible with a straightedge, compass, and p-sector, as well as characterizing the n-division points of other significant closed curves.

Summary Statement

This research project explores the field of n-division point constructibility with straightedge and compass constructions through three problems, characterizing the n-division points of the circle, tricuspoid, and Serret Curves.

Help Received

Worked with Dr. Simon Rubinstein Salzedo from Stanford who taught me much of the background for my research; Mr. Spenner, my science teacher, sponsored my project to Synopsys.