# CALIFORNIA STATE SCIENCE FAIR
# 2014 PROJECT SUMMARY

**Name(s)**
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**Project Number**
**S1416**

**Project Title**
## Characterizing the n-Division Points of Genus-0 Curves through Straight Edge and Compass Constructions

**Abstract**

**Objectives/Goals**
This project explores the field of constructibility of n-division points (points dividing a closed curve into n pieces of equal arc length) through straightedge and compass constructions, by furthering the work done by the two major theorems in this field - the Gauss-Wantzel Theorem and Abel's Theorem on the Lemniscate. Three major problems in this field were investigated: determining a closed form solution for the regular polygons that can be constructed with a straightedge, compass, and trisector; finding the values of n such that the n-division points of the tricuspoid can be constructed with a straightedge and compass; and seeking a generalization of Abel's Theorem to the entire family of Serret curves (a family of curves including the lemniscate whose arc lengths share properties).

**Methods/Materials**
Two major mathematical fields were used to algebraically represent the problem of constructibility. Field theory was used to characterize the figures, angles, and lengths that are constructible under a given set of conditions, and the theory of elliptic integrals was used to determine expressions for the n-division points of these curves. Three areas of mathematics were used in the proofs of the theorems. Galois theory was used to find a solution to the first problem, algebraic geometry was used to relate geometric problems of constructibility and n-division points to algebraic ones, and complex analysis was used to examine the elliptic integrals that characterized the arc length of these curves, particularly when solving the third problem.

**Results**
Three major results were obtained through the research. A closed-form solution for the values of n for which the n-division points of a circle can be constructed with a compass, straightedge, and trisector was found; a theorem was proved that for all integer n, the n-division points of the tricuspoid curve are constructible; and it was determined that with a compass and a straightedge, arbitrary arc lengths on any Serret Curve can be added, subtracted, and multiplied.

**Conclusions/Discussion**
These results represent the product of a year of investigation, however, additional work is being done to explore related problems in this field, such as examining the n-division points of a circle constructible with a straightedge, compass, and p-sector, as well as characterizing the n-division points of other significant closed curves.

**Summary Statement**
This research project explores the field of n-division point constructibility with straightedge and compass constructions through three problems, characterizing the n-division points of the circle, tricuspoid, and Serret Curves.

**Help Received**
Worked with Dr. Simon Rubinstein Salzedo from Stanford who taught me much of the background for my research; Mr. Spenner, my science teacher, sponsored my project to Synopsys.