CALIFORNIA STATE SCIENCE FAIR
2016 PROJECT SUMMARY

Name(s)

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Project Number

S1407

Project Title

The Algebra and Geometry of Quasicategories

Abstract

Objectives/Goals

The goal of my research was to study the quasi-2-categorical analogues of 1) abelian groups and spectra, which would generalize the notion of addition, and 2) operads and quasioperads, which would generalize the notion of a multiplication on a ring or a ring spectrum.

Results

I determined that there exists a consistent notion of stable quasi-2-categories which generalizes the notion of stable quasicategories that allows one to do basic homotopy-theoretic constructions with (symmetric monoidal) stable quasi-categories themselves. I also found that there exists a consistent notion of quasi-2-operads obtained by generalizing the notion of a fibration to the context of scaled simplicial sets, which encodes, like ordinary operads, the notion of a multiplication, but for objects of quasi-2-categories, and not just ordinary categories. These allow one to define generalizations of ordinary (commutative) rings and E_k-rings to the quasi-2-categorical world; the resulting theory can be applied to study an analogue of chromatic homotopy theory for stable quasicategories.

Conclusions/Discussion

This project sets the precedent for motivating the study of stable quasi-n-categories and quasi-n-categories for values of n>2, the foundation for which depends on the proof of an open problem in homotopy theory. More importantly, however, my research on "derived chromatic homotopy theory" and the notion of the derived moduli stack of elliptic curves could allow for a form of "derived tmf", which could potentially be extended not just to the n=2 case. This might allow for a deeper study of the relationship between homotopy theory and number theory.

Summary Statement

My research allowed for a generalization of the theory of commutative algebra using the notions of stable quasi-2-categories and quasi-2-operads which I developed, allowing me to describe a geometric approach to studying homotopy theory.

Help Received

Professor Marcy Robertson helped me a lot in teaching me about models of quasi-operads. The discussions in the homotopy theory chat room taught me a lot of interesting mathematics, which helped enrich the background knowledge for my project.