CALIFORNIA STATE SCIENCE FAIR
2017 PROJECT SUMMARY

Name(s)

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Project Number

S1521

Project Title

Signatures of Stable Multiplicity Spaces in Restrictions of Representations of Symmetric Groups

Abstract

Objectives/Goals

My goal was to find formulas for norms of orthogonal basis vectors in the multiplicity space of copies of an irreducible representation of S_{n-k} in the restriction of an irreducible representation of S_n, and for signatures counting how often the norms are negative.

Methods/Materials

I constructed a stable sequence of multiplicity spaces with fixed k and varying n. I used a basis of standard Young tableaux and applied a series of adjacent transpositions to their entries to find norm formulas in terms of n, then interpolated to arbitrary real values of n and studied the resulting signatures.

Results

I found explicit norm formulas, which give an immediate algorithm for computing signatures. I found that the norms are rational functions of n with positive integer roots coming in pairs that differ by 1, implying that the norm is always positive for sufficiently large or small n. I also found that the norm is the same for all basis vectors in the case where the partitions corresponding to the representation of S_n and S_{n-k} have the same first element. I also found explicit signature formulas in certain cases.

Conclusions/Discussion

The study of stability in representation theory by generalizing to arbitrary real or complex rank is fairly new and leads to rich but largely unexplored structures. My results give interesting information about certain combinatorial invariants in Deligne categories (generalizations of the representations of S_n to an arbitrary complex number), and they help give a better sense of how these categories behave, methods for studying them, and what related formulas might look like.

Summary Statement

I studied generalizations of the representations of the symmetric groups S_n to an arbitrary real number n, which gives richer structures than positive integer cases and provides information about stable properties of the representations.

Help Received

I was matched with the project by the PRIMES-USA program. The problem was suggested by Professor Pavel Etingof (MIT). Siddharth Venkatesh (MIT) helped me understand the necessary background and gave me advice throught the project. I found all the formulas and results myself.