Objectives/Goals
Force fields affect the work needed for an object to travel between two points in space. In this project, a particle traveled under the influence of five force fields (shown below) with only an engine force \( r(\text{engine})(t) \) controlling its path. The purpose of this project was to determine how to minimize the energy consumed by the engine as the particle traveled from the origin to an endpoint.

1) Earth's gravity only
2) Earth's gravity and air friction
3) Earth's gravity and a wind force
4) Gravity with two planets
5) Custom force field

Methods/Materials
The particle was a point mass of 800 kg. Three endpoints (500, 500, 500), (7500, 800, 600), and (90000, 60000, 40000) (meters) were selected. \( r(\text{field})(t) \) denotes the force field, and \( r(t) \) denotes the particle's path. \( r(\text{engine})(t) \) was calculated by subtracting \( r(\text{field})(t) \) from 800\( r''(t) \). For each endpoint in each field, about ten paths \( r(t) \) were selected. The paths involved constant velocity (control), polynomial, trigonometric, and exponential functions of \( t \), and \( t(\text{final}) \) had values between 10 and 1000 seconds. Using Mathematica, the total energy was calculated by adding the initial kinetic energy to the work integral \( W = \#\int_0^{t(\text{final})} r(\text{engine})(t) \cdot dr'(t) \).

Results
Energy was decreased for longer traveling times for all force fields. Trigonometric and exponential functions tended to produce significantly high energies. Under Force Fields 1, 2 and 3, a quadratic function of \( t \) with a negative leading term decreased energy. In Force Fields 4 and 5, several third degree polynomial paths with a negative leading term required less energy than using constant velocity. In Force Field 5, using a quartic term produced high energy, but using an appropriate sine wave and combining it with a linear or quadratic term can make the path more energy-efficient than the control.

Conclusions/Discussion
All the force fields showed that under the same traveling time, an appropriate deceleration from the beginning of the particle#s path required less energy than traveling at a constant velocity, which required less energy than high acceleration. The use of work integrals and kinetic energy in modeling the total energy consumption under several forces is essential in finding the minimal fuel required for a vehicle traveling between destinations. This may motivate newer methods of flying aircraft or generating electric power.

Summary Statement
Mathematica was used to calculate the total energy required for a particle to travel from the origin to three different endpoints under five different force fields for a variety of paths, and the paths that minimized energy were determined.

Help Received
Math teacher Mrs. Brown, physics teacher Mr. Harvie, advisor Mrs. Newman, and UCSD Physics professor Dr. David Brown helped explain concepts used in the project. Mother helped glue board and make Excel charts.