CALIFORNIA STATE SCIENCE FAIR
2011 PROJECT SUMMARY

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Project Number
31197

Project Title

A New Method to Prove Euler's Equation by Using the Lagrange Mean Value Theorem

Abstract

Objectives/Goals

Euler's Equation is beautiful formula combines five fundamental numbers in one and is generally considered to be the most elegant results in mathematics. Traditionally, we use four methods to prove it. We can use the Taylor series method which is a most popular way and can be found in our text books, the differential method, ordinary differential equations, or complex integrals. First, I enumerated the traditional methods. Then, I used the Lagrange Mean Value Theorem a new method to prove Euler's Equation. This method is clear and easy to understand. It is very helpful to understand both Euler's Equation and Lagrange mean value Theorem when learning calculus, especially for college and high school AP calculus courses.

Methods/Materials

My project was finding a new way to prove Euler's Equation by using the Lagrange Mean Value Theorem. First, I introduced and defined an auxiliary function f(x)=[e^(ix)]/(cos x + i sin x), I = (-∞, +∞), and I used the flowing steps to prove the Euler's Equation

1. Prove the Auxiliary Function f(x) = [e^(ix)]/(cos x + i sin x) is defined before we use it by proving that the denominator (cos x + i sin x) is not equal zero.
2. Compare differentiability of real functions and complex functions and took the derivative of the function f(x) = [e^(ix)]/(cos x + i sin x).
3. Using the Lagrange Mean Value Theorem, #If f#(x) =0 for all x in an interval (a, b), then f(x) is a constant c on (a, b)# and prove the constant c=1.

Results

Since f#(x) = 0 and f(x) = c, when x = 0, f(0) = 1, therefore f(x) = 1 and c = 1.
f(x) = [e^(ix)]/(cos x + i sin x) = 1, [e^(ix)] = (cos x + i sin x)
So when x = pi, cos(pi) = -1 and sin(pi) = 0.
We can conclude Euler#s identity e^[i(pi)]+1 = 0
Euler's Equation has been proven.

Conclusions/Discussion

This project is pure and theoretical mathematics. It uses the Lagrange Mean Value Theorem to prove Euler's Equation because it is clear and easy to understand it. This method is very helpful to understanding both Euler's Equation and Lagrange mean value Theorem when learning calculus.

Summary Statement

Using the Lagrange Mean Value Theorem to prove Euler's Equation

Help Received

None

Preliminary