CALIFORNIA STATE SCIENCE FAIR
2015 PROJECT SUMMARY

Name(s)

Timothy J. Varghese

Project Number

35255

Project Title

From Sums over Natural Numbers to Sums over Primes

Abstract

Objectives/Goals

Any sum over the naturals such as 1 + 2 + 3 . . . n = F(n) can be converted to a sum over the primes such as 2 + 3 + 5 + . . p_n = P(n) where p_n is the n-th prime. I started by proposing an Expand-Sum-Prune (ESP) heuristic in which P(n) is approximated as F (n ln n)/ ln n. ESP provides correct asymptotic results for sums of prime powers, duplicating a result of Salat-Znam. The goals of this project are:

1. To examine the hypothesis that ESP fails when any one term is too large a fraction of the whole sum.
2. To find new sums over primes never published earlier
3. When ESP fails, to find better summation methods.

Methods/Materials

1. Series: I studied several sums over primes including the alternating series (2 - 3 +5 -7 + ...), reciprocal sums (1/(2*3*5)+ 1 (3*5*7) +. . .), and sums of prime powers (2^2 + 3^2 + 5 ^2 + . . .)
2. All estimates were checked for accuracy using a Visual C program that uses the sieve of Eratosthenes to produce (and sum) all primes up to 1000000.

Results

1. Alternating sum: Consider A = 2 # 3 + 5. I provide a new estimate of |A| = 0.5 p_n with errors less than 2% for 500<n<78,401 by summing half the prime gaps using a modified ESP method. When my estimate was posed on Math Overflow (viewed 386 times, 14 votes +1 badge for "good question"), mathematicians felt my new result was "almost certainly true". However, using current sieve techniques they can only prove unconditionally that |A|< p_n /64. My method generalizes to alternating series of prime powers. I published a new series for alternating primes squared in the Online Encyclopedia on Integer Sequences (OEIS) as A240860.
2. Reciprocal sums: I prove that S=1/(2*3*5) + 1/ (3*5*7) + ... converges and 0.0474 < S < 0.0475, published in the OEIS as A242187 using a Bound-Reduce that applies to the infinite series for e.
3. Better estimates for sums of prime powers: I found a better approximation than the Salat-Znam estimate using a balancing constant c. I found experimentally that the best values of c are roughly 0.6 for prime sums, 0.7 for squared sums, and 0.9 for cubed sums
4. New estimates from old: I found a new asymptotic estimate for prime products two at a time with 2% error, added to the OEIS as A024447.

Conclusions/Discussion

The hypothesis that ESP method fails if any term dominates (limit of ratio of largest term to sum does not tend to zero) is supported by results.

Summary Statement

As in Alladin where the peddler promises new lamps for old, I seek new series over primes from old series over integers, and new formulas derived from formulas for integers

Help Received

Neil Sloane,head of OEIS helped refine hypothesis, Erich Bach (Wisconsin) helped make program efficient, Father helped with program. Robert Oliver (Stanford) gave valuable suggestions.