CALIFORNIA STATE SCIENCE FAIR
2015 PROJECT SUMMARY

Name(s)

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Project Number

35941

Project Title

Arrangements of Minors in the Totally Positive Grassmannian and Sturmfels' Triangulation

Abstract

Objectives/Goals

We discuss arrangements of equal minors in the totally positive Grassmannian. It was previously shown that arrangements of equal minors of largest value correspond to the simplices in the Sturmfels' triangulation. Here we investigate a major problem of arrangements of equal minors of m-largest value and find relationships between the minors and maximal simplices of certain cubical distance in the Sturmfels' triangulation.

Methods/Materials

To conduct this research, I used several mathematical tools. I used the combinatorial theory of sorted sets, Skandera inequalities, the theory of the Totally Positive Grassmannian, the graphs of triangulations, and general combinatorics and linear algebra knowledge.

Results

Several major results were obtained through this research. I defined two new notions- one was the notion of movements between maximal minor arrangements, and the other was the notion of cubical distance, which says that two vertices in the graph of the Sturmfels' triangulation are of distance one if they lie on the same hypercube of any dimension. I formulated and proved two theorems about the arrangements of second largest and third largest minors. Then I posed a conjecture for the general case which I proved in several subcases.

Conclusions/Discussion

I found and proved important properties and relationships between arrangements of minors in totally positive matrices and other important combinatorial objects. Besides completely describing the arrangements of minors of second and third largest value, I formed a conjecture for the general r-th largest minors in terms of the new notion, multidimensional cubical distance, that I introduced, and showed that the conjecture is true in several subcases. It seems that the rest of the conjecture may be within reach using methods similar to those used in my paper. The surprising relationships I found between totally positive matrices and the Sturmfels' triangulation could pave the way for future advances in physics, computer science, and mathematics. A paper containing these results has been accepted to FPSAC 2015, the premier combinatorial conference.

Summary Statement

I studied arrangements of equal minors in the Totally Positive Grassmannian, a combinatorial object which could provide insights into physics, computer science, and mathematics.

Help Received

Research mentor and advisor: Miriam Farber, graduate student at MIT

Preliminary